Binary black hole shadows, chaotic scattering and the Cantor set

Jake O. Shipley* and Sam R. Dolan†

School of Mathematics and Statistics, University of Sheffield, Hicks Building, Hounsfield Road, Sheffield, S3 7RH

Motivation

Using the **Event Horizon Telescope**, astronomers hope to generate high-resolution images of the black hole (BH) **shadow** at the centre of our galaxy in 2017.

Binary BH shadows can be understood by looking at **scattering of light** (null geodesics) around a pair of BHs.

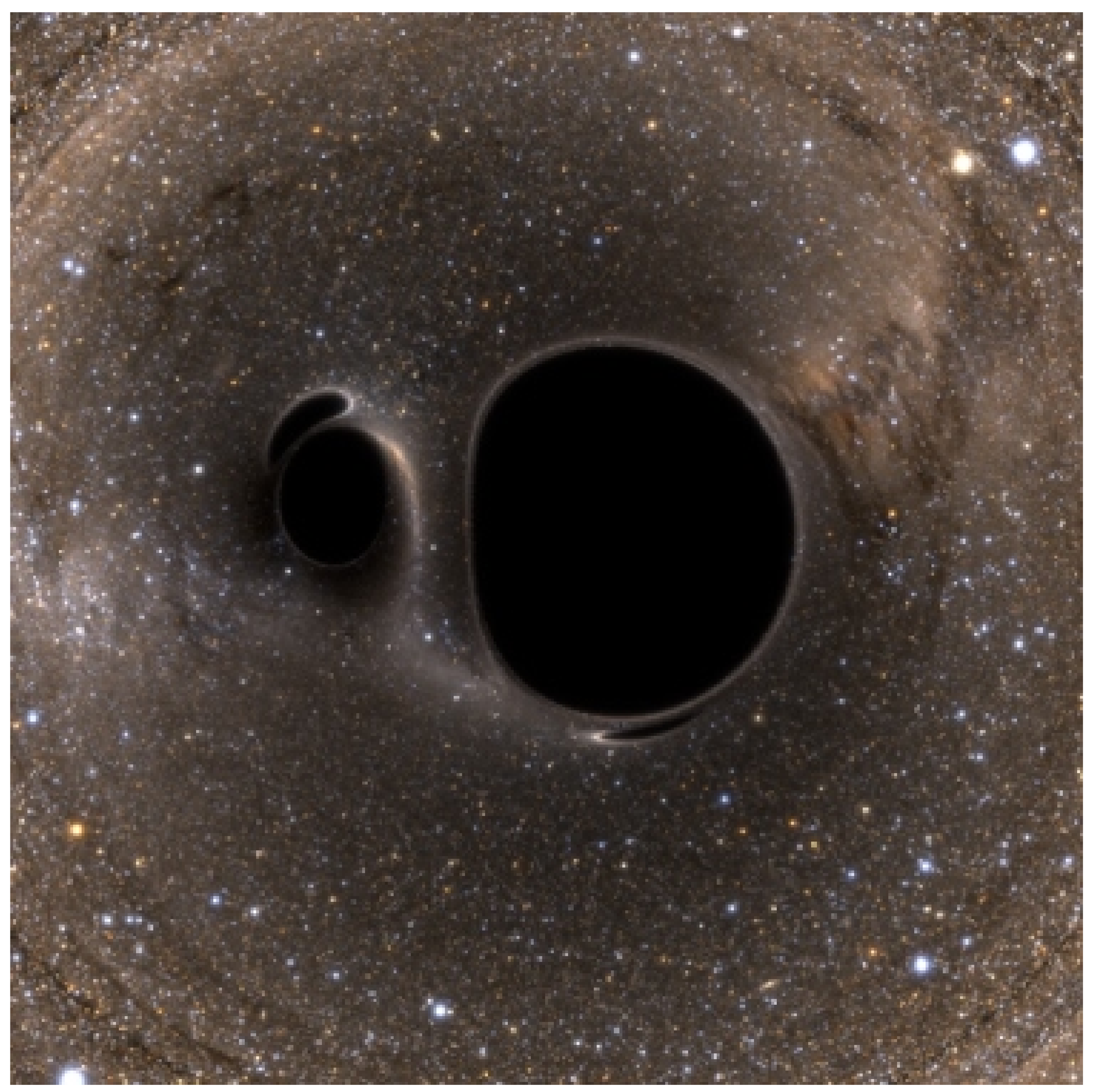


Figure 1: Binary BH shadow with self-similar eyebrow-like features. [Credit: SXS Lensing Group, www.black-holes.org]

Model

We use a **toy model** of two extremal BHs in static equilibrium. The Majumdar–Papapetrou metric reads

$$ds^2 = -U^{-2}dt^2 + U^2 d\mathbf{x}\cdot d\mathbf{x}, \qquad (1)$$

where

$$U = 1 + \frac{M_1}{\sqrt{\rho^2 + (z-a/2)^2}} + \frac{M_2}{\sqrt{\rho^2 + (z+a/2)^2}}, \qquad (2)$$

and a is the separation between the BHs.

The Hamiltonian for this system is

$$H = \frac{1}{2}\tilde{g}^{ab}p_a p_b = \frac{1}{2}\left(p_\rho^2 + p_z^2\right) + \frac{1}{2}\left(\frac{p_\phi^2}{\rho^2} - U^4\right). \qquad (3)$$

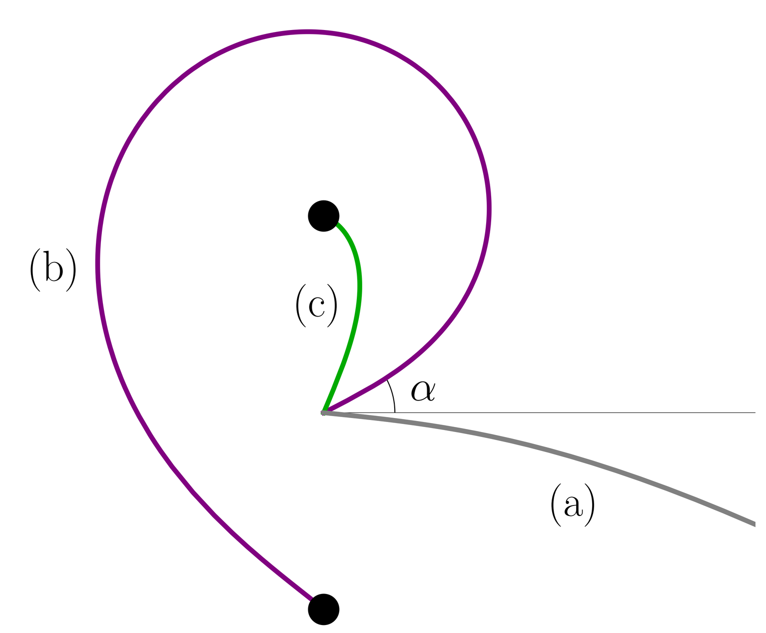
This is a **non-integrable 2D Hamiltonian system**. To analyse the scattering of light, we look at Hamilton's equations.

What is a black hole shadow?

Ray-casting approach: Null rays are traced away from the camera lens, backwards in time. The shadow is the region of the image taken by the camera where rays are traced back to a BH.

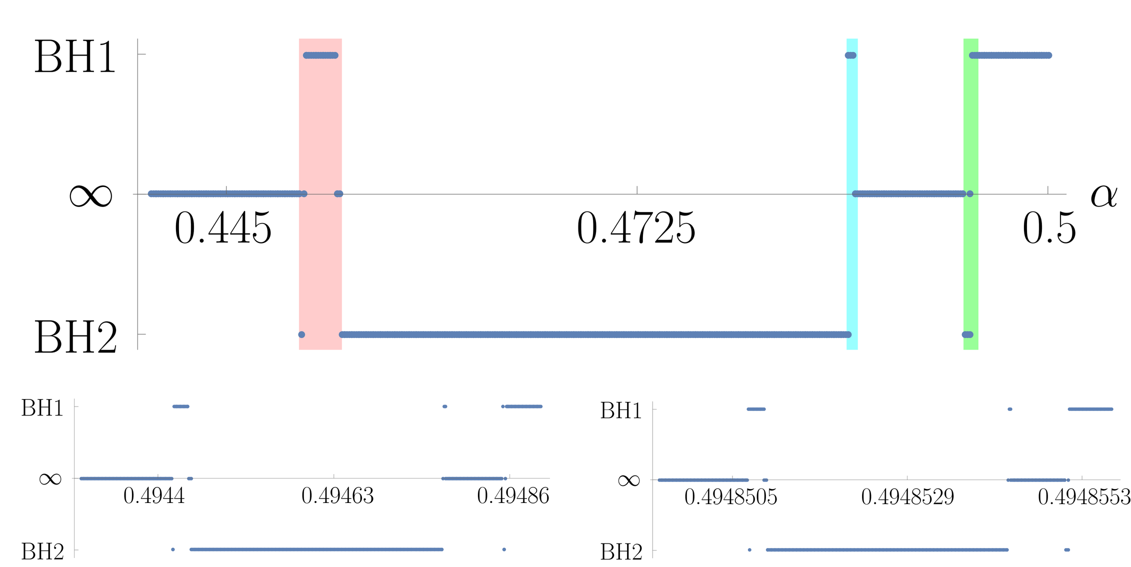
One-parameter scattering problem

For simplicity, we look at a 1-parameter scattering problem. Send null rays out from midpoint between BHs, varying initial angle of elevation from horizontal. Null geodesics can: **(a)** escape to ∞; **(b)** fall into lower BH; **(c)** fall into upper BH.



There is also a set of **unstable perpetual orbits** (not scattered or absorbed by the BHs). These lie on the interface between the absorbed and scattered rays.

The 1-parameter scattering problem admits a **1D shadow**. The structure of this 1D shadow allows us to understand 2D shadow images.

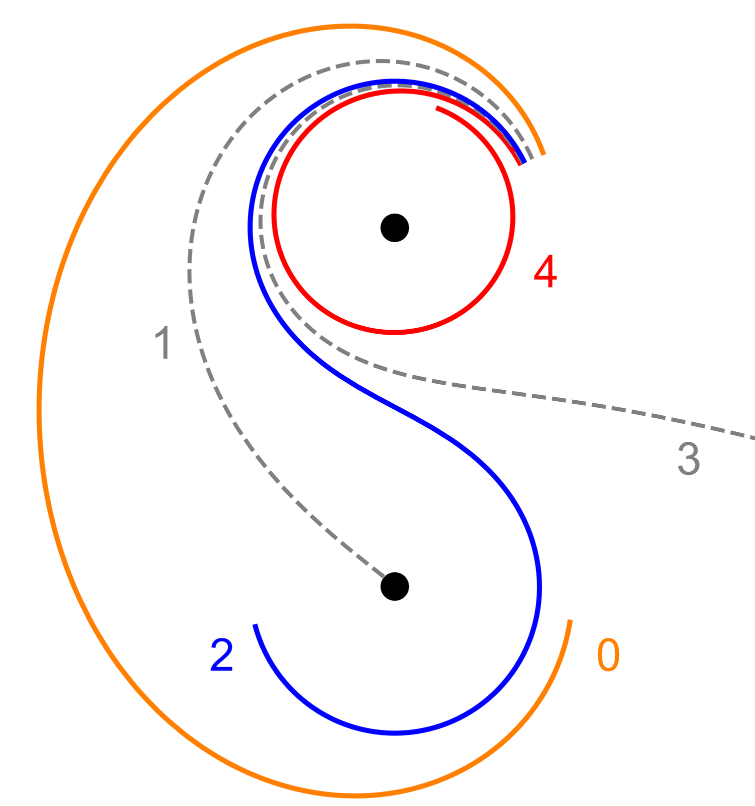


Zooming in on the region highlighted in green, we see **self-similarity** in the 1D shadow.

Symbolic dynamics

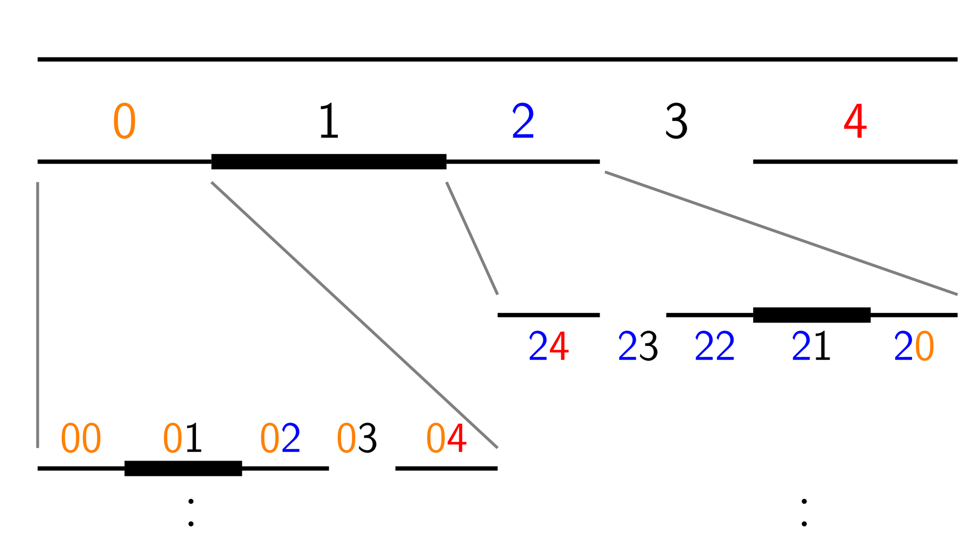
We have developed a **symbolic code** based on **'decision points'**. A geodesic which has travelled around a BH can:

0 travel around the other BH in the same sense;
1 fall into a BH (absorbed);
2 travel around the other BH in the opposite sense;
3 escape to ∞ (scattered);
4 travel around the same BH again.



Building the 1D shadow

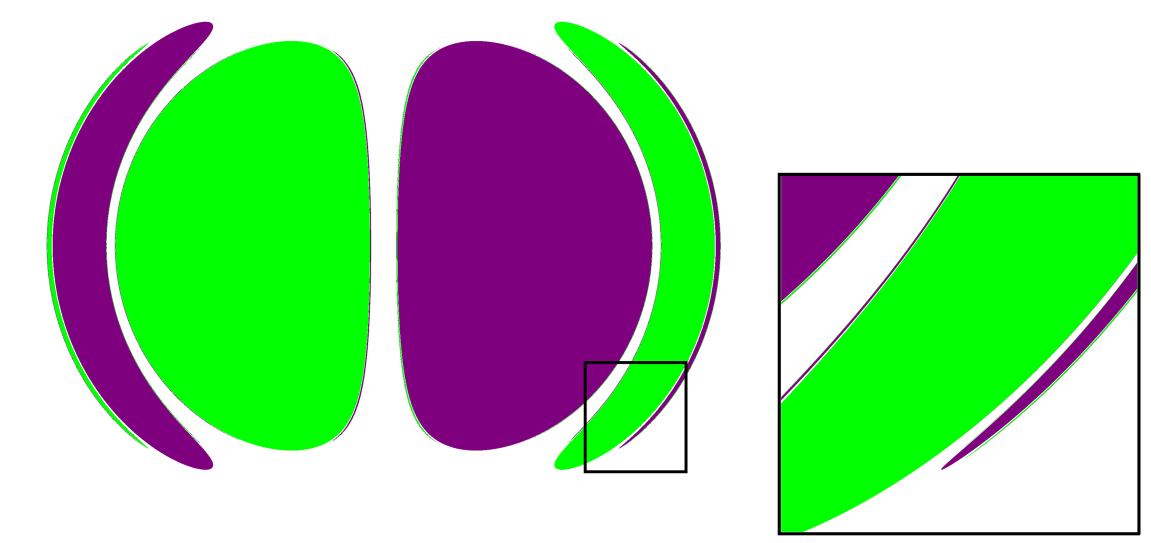
Using an iterative process, based on symbolic dynamics, we can construct 1D shadow. At each stage, remove intervals of initial data corresponding to decisions **1** and **3**. Iterate on the remaining intervals.

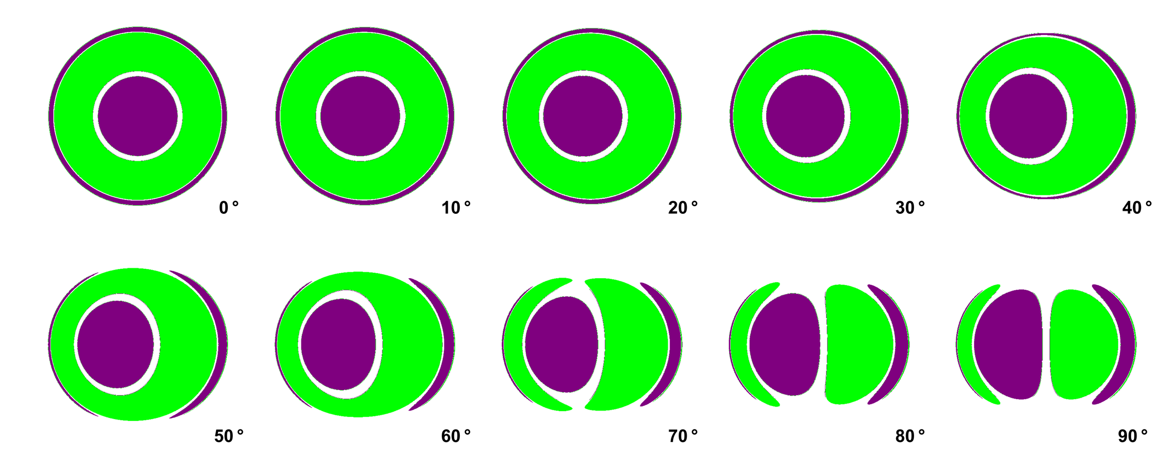


After infinitely many iterations, we are left with a **Cantor-like set** of perpetual orbits. The 1D shadow is the infinite union of open intervals, ending in the digit **1**.

2D shadows

The fractal structure of the 1D shadow is inherited by 2D shadows. We present a gallery of 2D shadows for different viewing angles. At 0° we look along axis connecting BHs. At 90° we look side-on at BHs.





In each case, main shadow images are surrounded by an infinite hierarchy of self-similar concentric ring-like structures or eyebrow-like features.

Conclusions

- Binary BH system exhibits **chaotic scattering**: it admits an infinite set of perpetual orbits as a **Cantor-like set** on the initial data.
- We have developed **symbolic dynamics** to describe **perpetual orbits**.
- We were able to construct the BH shadow using an iterative process based on **symbolic dynamics**.
- Binary BH shadows have **fractal** properties.

References

[1] J. O. Shipley & S. R. Dolan **arXiv:1603.04469**

* joshipley1@sheffield.ac.uk
† s.dolan@sheffield.ac.uk